

Indian Statistical Institute
B.Math. (Hons.) I Year
First Semester Backpaper Exam, 2006-2007
Probability Theory I

Time: 3 hrs

Date: -01-07

Max. Marks : 100

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1. An insurance company insured 2000 scooter drivers, 4000 car drivers, and 6000 truck drivers. The probability of an accident involving a scooter driver, car driver, truck driver are respectively 0.01, 0.03, 0.15. One of the insured persons meets with an accident. What is the probability that he is a scooter driver? [15]
2. 7 cards are drawn without replacement from a standard pack of 52 cards. Let X denote the number of aces drawn, and Y the number of kings drawn.
 - (i) Find the joint discrete density function of X, Y .
 - (ii) Find the marginal discrete density functions.
 - (iii) Are X and Y independent?

[10+6+4]

3. Let X, Y be independent random variables having Poisson distribution with parameters $\lambda_1 > 0, \lambda_2 > 0$ respectively
 - (i) Find the distribution of $X + Y$.
 - (ii) Find the conditional distribution of X given $X + Y = k$ for $k = 0, 1, 2, \dots$

[10+10]

4. (i) Let X be a nonnegative integer valued random variable having finite expectation. Show that

$$E(X) = \sum_{k=1}^{\infty} P(X \geq k).$$

- (ii) Let X be a positive absolutely continuous random variable. Show that

$$E(X) = \int_0^{\infty} P(X > x) dx$$

assuming that $E(X)$ exists.

[10+10]

5. Let X have the standard normal distribution. Find the probability density function of $Y = |X|$. [15]
6. Let X be an absolutely continuous random variable having an exponential distribution with parameter $\lambda > 0$. Find the moment generating function $m_X(t)$, indicating clearly for what t it exists. [10]